Acknowledgment

The authors are thankful for the valuable discussions held with Prof. Dr. David A. Rogers from North Dakota State University, Fargo, ND.

References

[1] D. Marcuse, "Interdependence of waveguide and material dispersion," *Appl. Opt.*, vol. 18, no. 7, pp. 2930–2932, Sept. 1979.
[2] P S M. Pires and D. A. Rogers, "Non-additivity of waveguide and material dispersion in single-mode step-index optical fibers: Exact analysis," in Proc. *URSI—Nat. Radio Sci. Meet.*, May 1982, pp. 22.
[3] P. S. M Pires, D. A. Rogers, E. J. Bochove, and R. F. Souza, "Prediction of laser wavelength for minimum total dispersion in single-mode step-index fibers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 131–140, Feb. 1982.
[4] C. T Chang, "Minimum dispersion in single-mode step-index optical fibers," *Appl. Opt.*, vol. 18, pp. 2516–2522, July 1979.
[5] ——, "Minimum dispersion at 1.55 μm for single-mode step-index fibers," *Electron. Lett.*, vol. 15, no. 23, pp. 765–767, Nov. 1979.
[6] C. R. South "Total dispersion in step-index monomode fibers," *Electron Lett.*, vol. 15, no. 13, pp. 394–395, June 1979.
[7] P. S. M Pires, "Analysis of dispersion in single-mode step-index optical fibers without the use of asymptotic expressions," M S.E.E thesis, State Univ. of Campinas, Brazil, Aug. 1980, (in Portuguese)
[8] M. Miyagi and S. Nishida, "An approximate formula for describing dispersion properties of optical dielectric slab and fiber waveguides," *J. Opt. Soc. Amer.*, vol 69, no. 2, pp. 291–293, Feb. 1979.
[9] R. A. Sammut, "Analysis of approximation for the mode dispersion in monomode fibers," *Electron Lett.*, vol. 15, no. 19, pp. 590–591, Sept 1979.
[10] P S. M Pires and D. A. Rogers, "Single-mode fiber design for minimum dispersion," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, no. 11, pp. 959–962, Nov. 1983.
[11] M. Abramowitz and I A. Stegun, *Handbook of Mathematical Functions*, U.S Dept. of Commerce, NBS, AMS-55, Dec. 1972.
[12] I W. Fleming, "Material dispersion in lightguide glasses," *Electron. Lett.*, vol. 14, no. 11, pp 326–328, May 1978.

On the Measurement of Noise Parameters of Microwave Two-Ports

MARIAN W. POSPIESZALSKI, SENIOR MEMBER, IEEE

***Abstract*—A novel procedure for determining the noise parameters of microwave two-ports is introduced. In this procedure, the computations necessary to find the noise parameters from the set of measurements of noise temperature (noise figure) are greatly simplified. The assessment of accuracy with which the noise parameters can be determined from a given set of measurement data is straightforward.**

I. Introduction

A typical noise parameter measurement setup is schematically shown in Fig. 1. The noise parameters of a device under test (DUT) and those of a receiver are represented by pairs of noise sources having correlation matrices [1]–[5]

$$[C_D]=\begin{bmatrix}\overline{e_{nD}^2} & \overline{e_{nD}i_{nD}^*}\\ \overline{e_{nD}^*i_{nD}} & \overline{i_{nD}^2}\end{bmatrix} \tag{1}$$

$$[C_R]=\begin{bmatrix}\overline{e_{nR}^2} & \overline{e_{nR}i_{nR}^*}\\ \overline{e_{nR}^*i_{nR}} & \overline{i_{nR}^2}\end{bmatrix} \tag{2}$$

Manuscript received June 5, 1985; revised November 14, 1985.
M W. Pospieszalski is with the National Radio Astronomy Observatory, Charlottesville, VA 22903. The National Radio Astronomy Observatory is operated by the Associated Universities, Inc, under contract with the National Science Foundation.
IEEE Log Number 8407189

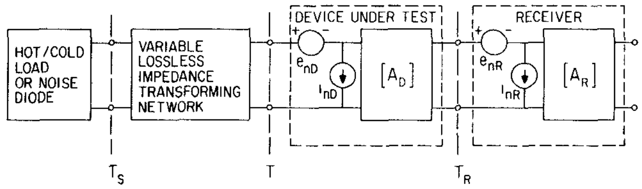

Fig. 1. A typical noise parameter measurement setup.

respectively. These matrices have to be Hermitian and nonnegative definite [3]–[5]. If the signal parameters of a DUT are given by chain matrix $[A_D]$, then the noise parameters of the cascade connection of the DUT and receiver given in a form of correlation matrix $[C]$ are [5]

$$[C]=[C_D]+[A_D][C_R][A_D]^{\dagger} \tag{3}$$

where the "dagger" designates the complex conjugate of the transpose of $[A_D]$ matrix. Matrix $[C]$ represents the noise parameters that can be determined at plane T (Fig. 1) by at least four noise temperature (noise figure) measurements for different values of source impedance as provided by the impedance transforming network. It is clear that if the noise parameters of a DUT are desired, the receiver contribution can be removed using (3), provided receiver noise parameters and device signal parameters are known.

The noise temperature T_n of any linear two-port is most commonly written in the following form [1], [2]:

$$T_n=T_{\min}+NT_0\frac{|Z_s-Z_{\mathrm{opt}}|^2}{R_sR_{\mathrm{opt}}}$$

$$=T_{\min}+NT_0\frac{|Y_s-Y_{\mathrm{opt}}|^2}{G_sG_{\mathrm{opt}}} \tag{4}$$

where

$$N=G_nR_{\mathrm{opt}}=R_nG_{\mathrm{opt}} \tag{5}$$

and

$T_{\min}$	minimum noise temperature,
$T_0=290$ K	standard temperature,
$Z_s=R_s+jX_s$	source impedance,
$Y_s=G_s+jB_s$	source admittance,
$Z_{\mathrm{opt}}=R_{\mathrm{opt}}+jX_{\mathrm{opt}}$	optimum source impedance,
$Y_{\mathrm{opt}}=G_{\mathrm{opt}}+jB_{\mathrm{opt}}$	optimum source admittance,
R_n	noise resistance,
G_n	noise conductance.

$T_{\min}$, R_{opt}, X_{opt}, G_n and $T_{\min}$, G_{opt}, B_{opt}, R_n are the sets of noise parameters equivalent to the correlation matrix $[C]$ (appropriate relations are given, for instance, in [2]). It has been shown that both $T_{\min}$ and the parameter N are invariant under transformation through lossless reciprocal two-ports connected to the input of a noisy two-port [2]. It also has been observed that for $T_{\min}$ and N to represent a physical two-port, the following inequality has to be satisfied [6]:

$$T_{\min}\leqslant 4NT_0. \tag{6}$$

This inequality (together with rather obvious conditions: $T_{\min}\geqslant 0, G_n\geqslant 0$ (or $R_n\geqslant 0$)) follows directly from the property that the correlation matrices have to be Hermitian and nonnegative definite. A simple physical interpretation of this inequality is

0018-9480/86/0400-0456$01.00 ©1986 IEEE

that two noise sources may not be more than 100-percent correlated.

In order to determine four noise parameters, it is now common practice to make a measurement of noise temperature for a number of different values of source impedance, greater than the minimum number of four. Then the influence of random errors may be reduced by a proper "fitting" procedure of a theoretical curve to measurement points. A number of computational methods have been proposed for this purpose [6]–[9]. These procedures seek the values of the noise parameters for which there is a minimum of an error function defined on the sets of measured values of noise temperature (or noise figure) and those predicted from (1). The definition of an error function and the choice of a set of noise parameters with respect to which this function is minimized differs from reference to reference. This minimization, however, has been performed with respect to all four noise parameters, which are varied independently, ignoring inequality (6). This occasionally results in the set of noise parameters that do not satisfy (6) and therefore do not represent the noise of any physical two-port [10].

The procedure proposed in this paper allows clear insight into this problem. Also, the errors in noise parameters determined from a given set of noise temperature measurements can be easily assessed.

II. Theory

It is evident from (4) that for any value of source resistance R_s, the minimum in noise temperature T_n occurs for the source reactance X_s or susceptance B_s equal to the optimum source reactance X_{opt} or susceptance B_{opt}. With the proper design of impedance transforming network, the condition for $X_s = X_{\mathrm{opt}}$ can be easily satisfied experimentally. In this case, (4) can be rewritten in the following form:

$$\frac{T_n}{T_0} = \left(\frac{T_{\min}}{T_0} - 2N\right) + N\left(\frac{R_s}{R_{\mathrm{opt}}} + \frac{R_{\mathrm{opt}}}{R_s}\right). \tag{7}$$

Choosing a new set of variables $y = T_n/T_0$, $x = R_s/R_{\mathrm{opt}} + R_{\mathrm{opt}}/R_s \geqslant 2$, (7) becomes

$$y = a + bx \tag{8}$$

where

$$a = \frac{T_{\min}}{T_0} - 2N$$
$$b = N. \tag{9}$$

The coefficients a and b are invariant under any lossless transformation at the input; consequently, (7) does not depend on the choice of the reference plane for the determination of source resistance R_s. This plane can be conveniently chosen for the realization of a number of different values R_{si} (or G_{si}) of source resistance (or conductance) for which the corresponding values T_{ni}^m of noise temperature are measured. Then the "fitting" algorithm for a number of measurements $n > 3$ can be reduced to finding the R_{opt} (or G_{opt}) for which the measured values of noise temperature T_{ni}^m for source resistances R_{si} (or conductances G_{si}) give the best fit to the straight line. For any assumed value of R_{opt}, the coefficients a and b of (8) can be found by linear regression, fitting the measured data with the mean square error of

$$E = \sqrt{\sum_{i=1}^{n} \left(T_{ni}^m - T_{ni}^c\right)^2} \tag{10}$$

where n is the number of measurement, T_{ni}^m and T_{ni}^c are measured and computed noise temperatures for given values of source resistance R_{si}. Clearly, the best estimation of R_{opt} (and, therefore, N and $T_{\min}$) for a given set of measurement is for error E assuming its minimum value.

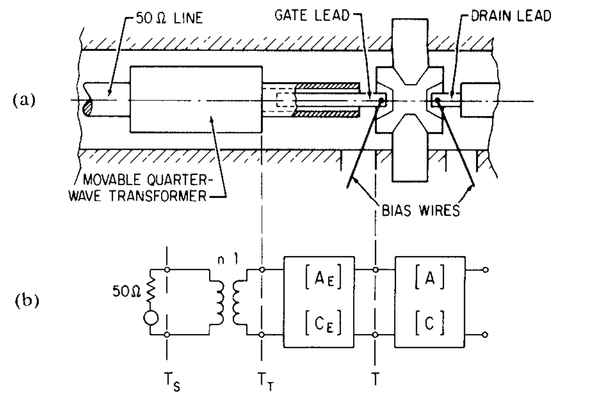

Fig. 2. (a) Construction of input impedance transforming network (b) Equivalent circuit at frequency at which the transformer is precisely quarter-wave.

III. Measurement Example

Simplicity and the relative advantages of the proposed procedure are demonstrated in the example of the noise measurement at cryogenic temperatures of experimental HEMT's developed at Cornell University [11]. The construction of the input impedance transforming network is shown schematically in Fig. 2(a) [12], [13]. The quarter-wave transformers are realized as movable slugs on a 50-Ω transmission line. The transistor gate lead, covered with teflon tubing placed inside the inner conductor of the 50-Ω line, forms a series stub of a reactance varying much more rapidly with frequency than the real part of source impedance or the noise parameters of a transistor. This assures that the minimum of measured noise temperature versus frequency occurs at a frequency at which $X_s = X_{\mathrm{opt}}$. The position of the transformer can then be easily adjusted so the minimum of noise temperature versus frequency coincides with the intended frequency of measurement at which the transformer is precisely quarter-wave.

In this case, the equivalent circuit shown in Fig. 2(b) is valid and at plane T_T: $X_{TS} = X_{T\mathrm{opt}} = 0$. If the graphical representation of noise parameters [14] at plane T_T is employed, the centers of the constant noise temperature (noise figure) circles lie on the real axis. Measuring T_n for number of transformers of different characteristic impedance positioned in the same plane yields the remaining noise parameters $T_{T\min}$, N_T, $R_{T\mathrm{opt}}$.

Any element of the equivalent circuit of the impedance transforming network that is not precisely known but is the same for all transformers can be treated as part of the two-port $[A_E]$, and therefore does not contribute any systematic errors to the determination of noise parameters at plane T_T.

Determination of the noise parameters at plane T is most simply done by using the relation

$$[C] = [A_E]^{-1}\{[C_T] - [C_E]\}[A_E]^{-1\dagger} \tag{11}$$

where the chain matrix $[A_E]$ and correlation matrix $[C_E]$ are of the passive, reciprocal two-port between planes T_T and T as indicated in Fig. 2. Equation (11) follows immediately from (3) rewritten for the circuit of Fig. 2. Of course, if this two-port is lossless, $[C_E] = 0$. In the presence of losses, $[C_E]$ can be easily determined from $[A_E]$ [15]. Formulas (3) and (11), as well as those for other interconnections of two-ports, and also conversions between different sets of noise parameters, lend themselves to computer implementation [16], [17].

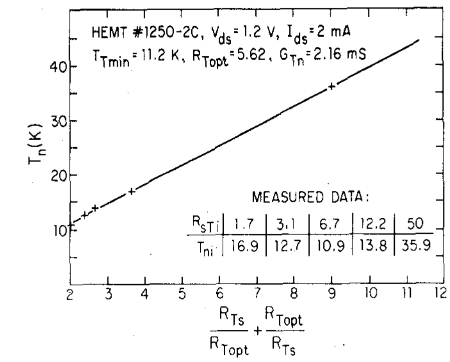

Fig. 3. Example of measured data (crosses) at 8.4-GHz frequency, corrected for receiver contribution, for cryogenically cooled (12.5 K) Cornell HEMT at reference plane T_T compared with minimum mean-square error E fit (solid line) resulting in noise parameters $T_{T\min}$, R_{Topt}, G_{Tn}.

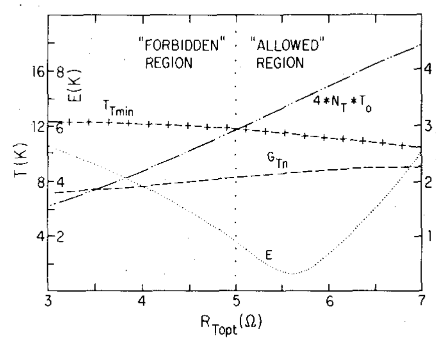

Fig. 4. Analysis of error in determination of noise parameters for the experimental data of Fig. 3. "Allowed" and "forbidden" regions for the set of noise parameters $T_{T\min}$, N_T, G_{Tn} are indicated.

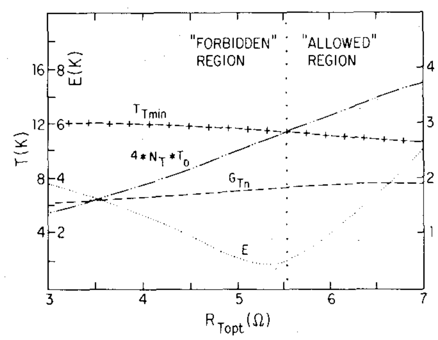

Fig. 5. Analysis of error in determination of noise parameters for the data of Fig. 3 altered in the following way: $T_{ni}(1.7) = 15.3$ K, $T_{ni}(50) = 32.3$ K. These alterations caused the minimum of E to occur in the "forbidden" region. The noise parameters for E assuming its minimum value do not satisfy the fundamental inequality (6).

An example of measured data for a cryogenically-cooled Cornell HEMT [11], [12], and its noise parameters determined at plane T_T by the described procedure and corrected for the receiver contributions are given in Fig. 3 (details of the measurement setup are given in [13]). The error analysis of this data is presented in Fig. 4.

The "allowed" and "forbidden" regions for noise parameters are indicated. It is clear that relatively small measurement errors may force the minimum of error function E into the "forbidden" region. It is demonstrated in Fig. 5, where first and last measurement results were altered to show how a minimum of E may occur in the "forbidden" region. This example demonstrates how easily, for a given set of measurements, the measurement error may lead to violations of fundamental inequality (6).

This is especially the case for very low-noise HEMT's or FET's in which the equivalent noise sources in drain and gate circuits are very closely correlated. If, therefore, in the course of measurement, only two from the noise parameters $T_{\min}$, N, R_{opt} can be dealt with accurately, then the inequality (6) provides a bound on the remaining parameter.

The accuracy with which the noise parameters at plane T can be determined from those at plan T_T is only dependent on the accuracy with which the signal parameters of two-port $T_T - T$, given for instance by $[A_E]$, are known. In most practical situations, this network can be considered lossless, which preserves the values of $T_{\min}$ and N. This, therefore, may cause the errors only in the values of R_{opt} and X_{opt}.

IV. Conclusions

The novel measurement procedure of the noise parameters of microwave two-ports has been presented. Not only are the computations simplified but, at different steps of the proposed procedure, the effect of measurement errors on the values of different noise parameters can be easily analyzed.

Acknowledgment

The many helpful discussions held with Drs. S. Weinreb and J. Granlund of NRAO during the course of this work are greatly appreciated.

References

[1] IRE Subcommittee 7.9 on Noise: "Representation of noise in linear two-ports," *Proc. IEEE*, vol. 48, pp. 69–74, Jan. 1960.
[2] J. Lange, "Noise characterization of linear two-ports in terms of invariant parameters," *IEEE J. Solid-State Circuits*, vol. SC-2, pp. 37–40, June 1967.
[3] H. Rothe and W. Dahlke, "Theory of noisy four poles," *Proc. IRE*, vol. 44, pp. 811–818, June 1956.
[4] H. Haus and R. B. Adler, "Optimum noise performance of linear amplifiers," *Proc. IRE*, vol. 46, pp. 1517–1533, Aug. 1958.
[5] H. Hillbrand and P. Russer, "An efficient method for computer-aided noise analysis of linear amplifier networks," *IEEE Trans. Circuits Syst.*, vol. CAS-23, pp. 235–238, Apr. 1976.
[6] W. Wiatr, "A method of estimating noise parameters of linear microwave two-ports," Ph.D. dissertation, Warsaw Tech. Univ., Warsaw, Poland, 1980 (in Polish).
[7] R. Q. Lane, "The determination of the device noise parameters," *Proc. IEEE*, vol. 116, pp. 1461–1462, Aug. 1969.
[8] G. Caruso and M. Sannino, "Computer-aided determination of microwave two-port noise parameters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 639–642, Sept. 1978.
[9] M. Mitama and H. Katoh, "An improved computational method for noise parameter measurement," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 612–615, June 1979.
[10] M. W. Pospieszalski and W. Wiatr, "Comment on design of microwave GaAs MESFET's for broad-band, low-noise amplifier," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, p. 194, Jan. 1986.
[11] L. H. Camnitz, P. A. Maki, P. J. Tasker, and L. F. Eastman, "Submicrometer, quantum well HEMT with an Al_3 Ga_7 As," in *Proc. 1984 Int. Symp. GaAs Related Compounds* (Biarritz, France) Sept. 1984, to be published in *Inst. Phys. Conf. Series*.
[12] S. Weinreb and M. Pospieszalski, "*X*-band noise parameters of HEMT devices at 300 K and 12.5 K," in *Proc. 1985 Int. Microwave Symp.* (St. Louis, MO), June 1985; pp. 539–542.
[13] M. Pospieszalski, "Low-noise, cryogenically cooled, 8.0–8.8-GHz amplifiers," National Radio Astronomy Observatory Electronics Div. Internal Rep. no. 254, Dec. 1984.
[14] H. Fukui, "Available power gain, noise figure, and noise measure of two-ports and their graphical representations," *IEEE Trans. Circuit Theory*, vol. CT-13, pp. 137–142, 1966.
[15] R. Q. Twiss, "Nyquist's and Thevenin's theorems generalized for nonreciprocal linear networks," *J. Appl. Phys.*, vol. 26, no. 5, pp. 599–602, May 1955.
[16] D. L. Fenstermacher, "A computer-aided analysis routine including optimization for microwave circuits and their noise," National Radio Astronomy Observatory Electronics Div. Internal Rep. no. 217, July 1981.
[17] J. Granlund, "FARANT on the HP9816 computer," National Radio Astronomy Observatory Electronics Div. Internal Rep. no. 250, Aug. 1984.